TECHNISCHE UNIVERSITÄT WIEN

# Solution approaches to a flexible job-shop scheduling problem with a quality control as stochastic element

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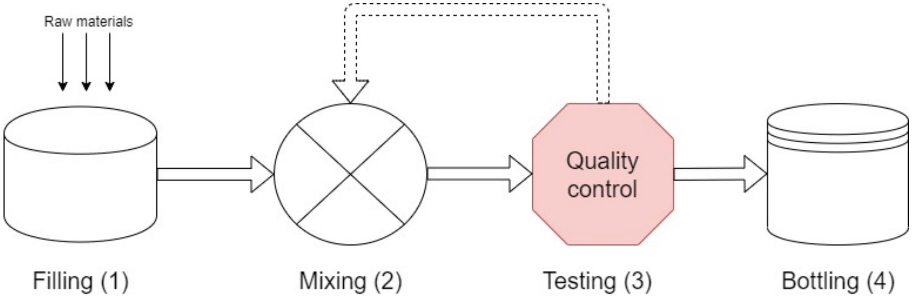
# Production Process



$$\min \ C^{\max}$$

s.t.

$$C^{\max} \geq C_{j,l_j}, \ \forall j \in J$$

$$\sum_{h=0}^{n} \sum_{z=1}^{z_h} \sum_{i=1}^{m} x_{j,l,h,z,i} = 1, \ \forall j \in J, \ l = 1, \ldots, l_j$$

$$\sum_{h=0}^{n} \sum_{z=1}^{z_h} x_{j,l,h,z,i} \leq \alpha_{j,l,i}, \ \forall j \in J, \ l = 1, \ldots, l_j, \ i \in M$$

$$\sum_{j=1}^{n} \sum_{l=1}^{l_j} \sum_{i=1}^{m} x_{j,l,h,z,i} \leq 1, \ \forall h \in J, \ z = 1, \ldots, z_h$$

$$\sum_{j=1}^{n} \sum_{l=1}^{l_j} x_{j,l,0,1,i} \leq 1, \ \forall i \in M$$

$$\sum_{j=1}^{n} \sum_{l=1}^{l_j} x_{j,l,h,z,i} \leq \sum_{j=0}^{n} \sum_{l=1}^{l_j} x_{h,z,j,l,i},$$
$$\forall h \in J, \ z = 1, \ldots, z_h, \ i \in M$$

$$C_{j,1} \geq p_{j,1}, \ \forall j \in J$$
$$C_{j,l} \geq C_{j,l-1} + p_{j,l}, \ \forall j \in J, \ l = 2, \ldots, l_j$$
$$C_{j,l} \geq C_{h,z} + \sum_{i=1}^{m} (x_{j,l,h,z,i} \cdot set_{j,h,i}) + p_{j,l}$$
$$- L \cdot \left(1 - \sum_{i=1}^{m} x_{j,l,h,z,i}\right),$$
$$\forall j \in J, \ l = 1, \ldots, l_j, \ h \in J, \ z = 1, \ldots z_h$$

$$\forall j, h \in J, \ j \neq h:$$
$$(C_{h,1} - p_{h,1}) - (C_{j,1} - p_{j,1}) + L \cdot (1 - a_{j,h}) \geq 0$$
$$(C_{j,1,s} - p_{j,1}) - (C_{h,1} - p_{h,1}) + L \cdot a_{j,h} \geq 0$$
$$a_{j,h} + a_{h,j} \leq 1$$
$$(C_{j,1} - p_{j,1}) - C_{h,z_h} + L \cdot (1 - b_{j,h}) \geq 0$$
$$C_{h,z_h} - (C_{j,1} - p_{j,1}) + L \cdot b_{j,h} \geq 0$$

$$\forall j, h \in J, \ j \neq h:$$
$$w_{j,h} - a_{j,h} - b_{j,h} \leq 0$$
$$a_{j,h} - w_{j,h} \leq 0$$
$$b_{j,h} - w_{j,h} \leq 0$$
$$\sum_{h \in J, j \neq h} w_{j,h} \geq n - v^{\max}, \ \forall j \in J$$
$$d_{j,v} + d_{h,v} \leq w_{j,h} + w_{h,j}, \ \forall v \in V, \ j, h \in J, \ j \neq h$$
$$\sum_{v=1}^{v^{\max}} d_{j,v} = 1, \ \forall j \in J$$

$$x_{j,l,h,z,i} \in \{0, 1\},$$
$$\forall i \in M, \ j, h \in J_0, \ l = 1, \ldots, l_j, \ z = 1, \ldots, z_h$$
$$a_{j,h}, \ b_{j,h}, \ w_{j,h} \in \{0, 1\}, \ \forall j, h \in J$$
$$d_{j,v} \in \{0, 1\}, \ \forall j \in J, \ v = 1, \ldots, v^{\max}$$
$$C_{j,l} \geq 0, \ \forall j \in J, \ l = 1, \ldots, l_j$$
$$C^{\max} \geq 0$$

# Uncertainty in the Model

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## Consequences

- uncertain number of mixing and testing steps
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## Representation of Uncertainty

- operation plan with maximum number of repetitions
  - stochastic process times for mixing and testing
  - discrete stochastic variables
  - two possible values
    - $= 0$ ... operation does not take place
    - $> 0$ ... operation takes place with process time $> 0$

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## Deterministic *Worst Case*

- solution under *worst case* assumption for all stochastic variables

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## Degree of Robustness

- reduce loss in optimality
- only a certain number $\Gamma$ of stochastic process times will deviate from nominal value
- *protection function*

$$\max_{\substack{S\cup\{t\},\ S\subseteq J,\\ |S|=\lfloor\Gamma\rfloor,\ t\in J\setminus S}} \left\{ \sum_{j'\in S} \sum_{\substack{(j',l')\in P,\\ (j',l')\neq(h,z)}} \rho_{j,l,h,z,j',l',i} \cdot p_{j',O_{j',l'}} + (\Gamma - \lfloor\Gamma\rfloor) \sum_{\substack{(t,l')\in P,\\ (t,l')\neq(h,z)}} \rho_{j,l,h,z,t,l',i} \cdot p_{t,O_{t,l'}} \right\}$$

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## Optimality

- minimize **expected** make span over all scenarios
  - weighted by the scenario's probability of occurrence

$$\min \ \mathbb{E}(C^{\max}) = \sum_{s \in S} \omega_s C_s^{\max}$$

$s \in S$ . . . set of all scenarios

$\omega_s \in [0, 1]$ . . . probability for scenario $s$

$C_s^{\max}$ . . . actual make span for scenario $s$

# Reactive Approach

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- monitor ongoing production
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## Idea

- monitor ongoing production
- adapting the plan after each quality control

## Algorithm

- starting solution
  - deterministic model
- plan for each possible outcome of the next quality control
  - stochastic model
  - *model predictive*
    - look ahead only a certain number of quality controls

# Thank you very much!

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